Densification of the IHRF in Denmark, The Faroe Islands, and Greenland

Hergeir Teitsson, Laura Sánchez, and René Forsberg

Abstract

The International Association of Geodesy (IAG) introduced and defined, in 2015, the International Height Reference System (IHRS) as the conventional reference system for the global physical height determination. Following the conventions for the realisation of the IHRS, i.e. the determination of the International Height Reference Frame (IHRF), we utilize the existing GNSS reference stations in Denmark, The Faroe Islands, and Greenland to determine a local densification of the IHRF in these regions. The physical heights of these Danish, Faroese and Greenlandic GNSS reference stations have been transformed from the local Danish, Faroese, and Greenlandic height systems, DVR90, FVR09 and GVR16, respectively, to geopotential numbers and normal heights referring to the IHRF. The offset to the IHRF is found to be −44.0 ± 1.9 cm, −59.8 ± 5.1 cm and − 54.1 ± 11.3 cm for the DVR90, FVR09 and GVR16, respectively. This transformation relies on the existing precise local (quasi-)geoid models. This contribution describes the applied procedures in the IHRF densification and discusses the quality assessment of the results.

Keywords

Global unified vertical reference system · IHRF densification · International Height Reference Frame (IHRF) · World height system

1 Introduction

A globally unified height system is important for consistent, homogeneous, and long-term stable observations in the Earth system, e.g. monitoring sea level change and other changes to climate (Ihde et al. 2017). Most countries use individually constructed height systems, which are local or regional, based on local sea level, and using different types of heights, with discrepancies to adjacent height systems (Sánchez et al. 2021a). In 2015 the International Association of Geodesy (IAG) defined the IHRS as the global standard for physical height determination (IAG Resolution No. 1 (2015), Drewes et al. 2016).

The IHRS definition states that the vertical reference level is the equipotential surface with the conventional geopotential value $W_{0,\,IHRF} = 62636853.4\ \mathrm{m^2/s^2}$. The vertical coordinate is the geopotential number $C(P)$, i.e. the difference between the potential $W(P)$ of the Earth's gravity field at a point P and the conventional W_0. The spatial reference of P is given in the International Terrestrial Reference System (ITRS, Petit and Luzum 2010), and the coordinates should be related to the mean tide system.

The realisation of the IHRS is the IHRF and it will be implemented by a well-distributed global core network. National and regional densifications of the IHRF are necessary to provide the most optimal height system unification (Sánchez et al. 2021a) and to provide an easy access to the IHRF. The aim of the article is to show the whole process

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of implementing the IHRF densification in the Kingdom of Denmark, i.e. Denmark, the Faroe Islands, and Greenland. From the choice of stations, acquiring coordinates in the International Terrestrial Reference Frame (ITRF, Altamimi et al. 2023) and gravimetric quasigeoids for the countries in the Kingdom of Denmark. Then, the determination of geopotential numbers in the IHRF and how it can differ, depending on the correction of the zero-degree term (ζ_0). An attempt at assessing the accuracy of the results is made by comparing to geopotential numbers from the global gravity models (GGM) EGM2008 (Pavlis et al. 2012, 2013) and XGM2019e (Zingerle et al. 2020), which is the alternative option to using the local/regional gravimetric quasigeoid.

Section 1 introduces the background and purpose of the article. The densification stations and gravimetric quasigeoid models are described in Sect. 2. The method of computing the IHRF geopotential numbers for the densifications is described in Sect. 3, both generally and the differences in computation for the densifications of the three countries in the Kingdom of Denmark. The results are shown and analysed in Sect. 4, followed by a discussion of the results in Sect. 5. The article is concluded in Sect. 6.

2 Data

2.1 Densification Stations

The Danish reference network consists of 14 Continuously Operating Reference Stations (CORS) distributed across Denmark (DK-net), providing Global Navigation Satellite System (GNSS) data. All of the CORS stations are co-located with absolute gravity stations. The Faroe Islands geometric reference network consists of 4 CORS-GNSS stations at locations distributed across The Faroe Islands (FO-net). The Greenlandic geometric reference network consists i.a. of 64 CORS-GNSS stations at 60 locations all around the coast of Greenland, which continuously monitor the land uplift on the island. These are called GNET, and 59 of the stations are included in the densification. The GNSS data from all stations is made available by the Danish Agency of Data Supply and Infrastructure (SDFI 2023) The tide system of the ITRF coordinates is assumed to be the tide-free system, as this is convention of the ITRF (Altamimi et al. 2023). In both Denmark and Greenland some CORS stations are not included, mainly because these were closely located to other CORS stations. In Denmark the CORS stations not co-located with absolute gravity were not included.

The convention of the IHRF refers to ITRF2014, epoch 2021.04. The ITRF coordinates of the stations are transformed to ITRF2014 (Altamimi et al. 2016) from other ITRF realisations, when necessary, with the standard seven-parameter transformation, which involves three shifts of the origin, three rotations and one scale

$$\begin{bmatrix} XS \\ YS \\ ZS \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} Tx \\ Ty \\ Tz \end{bmatrix} + \begin{bmatrix} D & -Rz & Ry \\ Rz & D & -Rx \\ -Ry & Rx & D \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}, \tag{1}$$

where X, Y, Z are the coordinates in e.g. ITRF2020, and XS, YS, ZS are the coordinates in ITRF2014. The transformation parameters Tx, Ty, Tz, Rx, Ry, Rz and D are derived from already published parameters in IERS Technical Notes and Annual Reports (ITRF-IGN 2023). Prior to analysis, positions and corrected using the simple linear model

$$P(t) = P(t_0) + \dot{P}(t - t_0) \tag{2}$$

converting the parameters P in eq. (X) from the reference epoch t_0, to the desired epoch t. The rate parameter $\dot{P}$ is also made available by the IERS.

2.2 Gravimetric Quasigeoid Models

The three countries in the Kingdom of Denmark have three different physical height systems and some details for each can be seen in Table 1.

For the Danish stations the *NKG2015 gravimetric quasigeoid* is used (Ågren et al. 2016), which is publicly available on the webpage of the International Service of the Geoid (Reguzzoni et al. 2021). It refers to the GRS80 ellipsoid (Moritz 2000) and to $W_{0,\,IHRF}$, but is shifted with a 1-parameter fit to GNSS-levelling height anomalies in the Nordic and Baltic countries ($\Delta h_{zero \rightarrow non-tidal}$ from Ekman 1989). Figure 1 shows the location of the DK-net stations on top of a plot of the NKG2015 gravimetric quasigeoid.

The physical height system in The Faroe Islands is based on the hybrid geoid FOGEOID2012, which in turn is based on the *FOGEOID2010 gravimetric quasigeoid* (Forsberg 2010, 2011) fitted to the local GNSS-levelling data existing over the region. The FOGEOID2010 was to computed as a quasigeoid. The quasigeoid model is not available online, but can be obtained from the Faroese Mapping Authority or DTU Space. Figure 2 shows the location of the FO-net stations on top of a plot of the FOGEOID2010 gravimetric quasigeoid.

The physical height system in Greenland is based on the geoid GGEOID16, defining a new geoid-based height system in Greenland, replacing some 78 local mean sea level (MSL) datums for individual towns and settlements. GGEOID16 is based on a gravimetric quasigeoid computation, followed by a geoid-quasigeoid separation correction (Forsberg 2016). The gravimetric quasigeoid can be made available from DTU Space or SDFI on request. The fit to Greenlandic GNSS-levelling in towns and settlements is rather poor, since local

Table 1 Details and specifications for the national height systems and gravimetric quasigeoid models used in the determination of the geopotential numbers in the IHRF

	Denmark	The Faroe Islands	Greenland
Physical height system	DVR90	FVR09	GVR16
Gravimetric quasigeoid model	NKG2015quasigeoid (epoch 2000)	FOGEOID2010	GGEOID16
Resolution of grav. Quasigeoid	1 km	0.5 km	2 km
No. stations	14	4	59
Reference ellipsoid	GRS80	GRS80	GRS80
GGM	GO_CONS_GFC_2_DIR_R5(Bruinsma et al. 2013)	EGM2008	EIGEN-6C4(Förste et al. 2014)
Uplift correction	NKG2016LU to epoch 2021.04(Vestøl et al. 2019)	Not applied	Not applied
1-p transf. Correction	$-(x_1 + \Delta h_{zero \rightarrow non-tidal}), x_1 = -0.4874\ m$	Not necessary	Not necessary
Permanent tide system	Zero-tide	Tide-free	Zero-tide
Bias to local GNSS-levelling ζ	−48.00 (after 1-p transf. and uplift correction)	−7.00 cm	−15.60 cm (shift to Nuuk MSL)
RMS with local GNSS-levelling ζ	1.70 cm	5.00 cm	10.00 cm

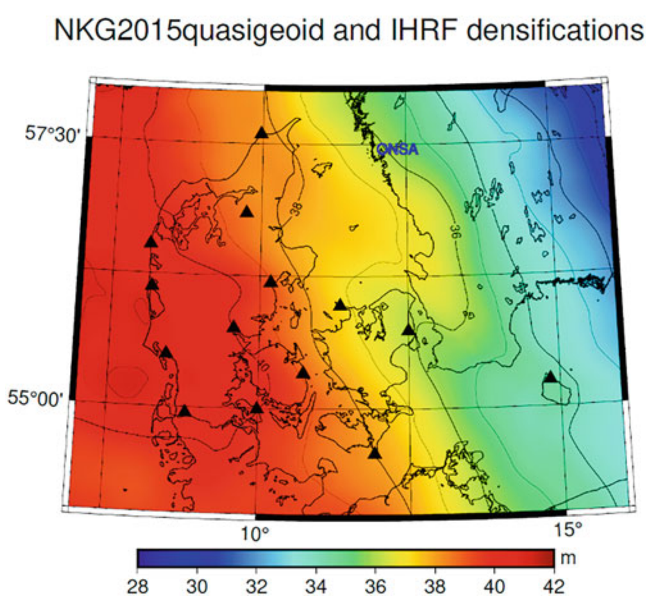

Fig. 1 The figure shows the local gravimetric quasigeoid NKG2015 over Denmark with 1 m contours. The locations of the 14 Danish IHRF densification network stations are shown as black triangles. The location of the IHRF global core station ONSA is indicated by the blue label

MSL systems are in use. Figure 3 shows the location of the GNET stations on a plot of the GGEOID16 gravimetric quasigeoid.

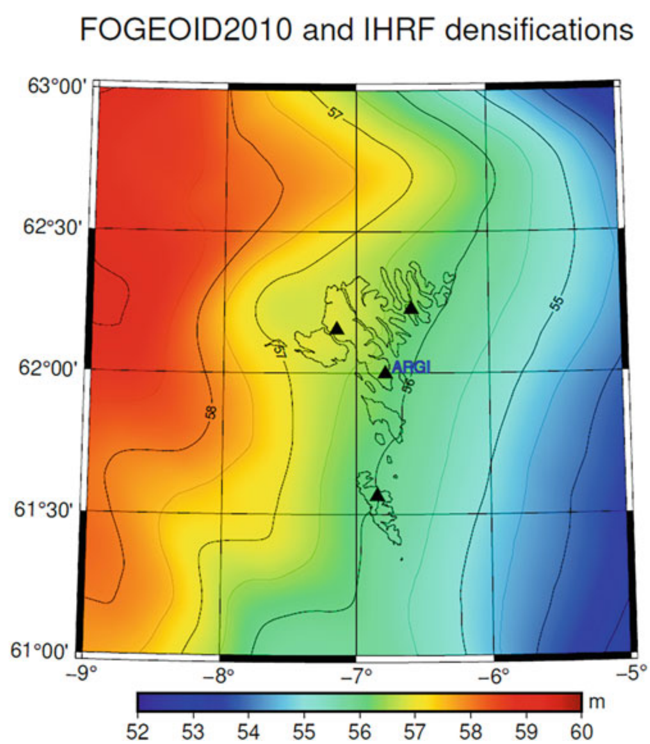

Fig. 2 The figure shows the local gravimetric quasigeoid FOGEOID2010 of The Faroe Islands, with 0.5 m contour lines. The locations of the 4 Faroese IHRF densification network stations are shown as black triangles on the plot. The location of the IHRF global core station ARGI is indicated by the blue label

3 Method

3.1 ζ_0-Corrections

The method for computing the gravity potential values at the stations will be outlined below. A detailed description can be found in Sánchez et al. (2021a).

The conventions of the IHRF prescribe the following conditions:

– The geopotential numbers must only come from the solution of the GBVP, i.e. no influence on the gravity potential values from GNSS-levelling data.
– The reference ellipsoid should be the GRS80.
– The geopotential numbers in IHRF are given in the mean-tide system.

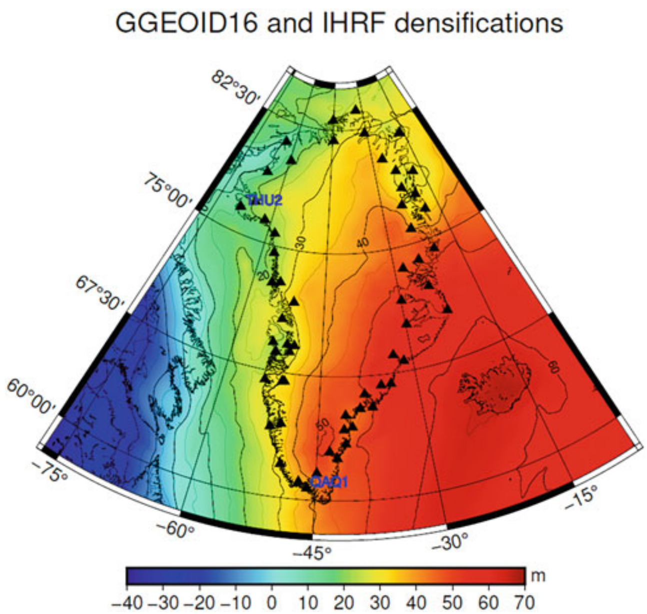

Fig. 3 The figure shows the local gravimetric quasigeoid GGEOID16 of Greenland, with 5 m contours. The locations of the 59 Greenlandic IHRF densification network stations are shown as black triangles on the plot. The location of the IHRF global core stations, THU2 and QAQ1 are indicated by the blue label

The realization of IHRS requires the determination of the geopotential numbers at the GNSS reference stations. The geopotential number C at a station P in the IHRF is defined as

$$C(P) = W_0 - W(P), \tag{3}$$

where $W_0 = W_{0,\,IHRF}$. Thus, we need to infer the IHRF potential values $W(P)$ at a station P and this depends on the solution of the GBVP, i.e. if the solution is the geoid or the quasigeoid. With Molodensky's theory of solving the GBVP the potential is determined at a point P on the surface of the Earth, which makes the determination of $W(P)$ and subsequently $C(P)$, rather straightforward. We work with the following solution of the GBVP (cf. Eq. 8–57, Hofmann-Wellenhof and Moritz 2006)

$$\zeta_{gravimetric} = \zeta_0 + \frac{R}{4\pi\gamma_0}\iint\limits_{\sigma}\left[\Delta g - \frac{\partial \Delta g}{\partial h}\left(h - h_p\right)\right] S\left(\psi\right) d\sigma, \tag{4}$$

where ζ_0 is the zero-degree term, R is the radius of the earth, γ_0 is the normal gravity at the ellipsoid, Δg is the free-air gravity anomaly, h is the ellipsoidal height, h_p is the ellipsoidal height at a computation point P, and $S(\psi)$ is Stokes function. ζ_0 can be regarded as the distance between geoid with the geopotential W_0 and the reference ellipsoid with the normal potential U_0. It can be computed by the generalization of Bruns' formula (cf. eq. (2–182), Heiskanen and Moritz 1967)

$$\zeta_0 = \frac{GM_{GGM} - GM_0}{r\gamma} - \frac{W_0 - U_0}{\gamma}, \tag{5}$$

where GM_{GGM} is the geocentric gravitational constant of the GGM, GM_0 is the geocentric gravitational constant of the reference ellipsoid (normal gravity field), r is the radial distance, γ is normal gravity, W_0 is the gravity potential of the geoid, and U_0 is the normal potential at the surface of the reference ellipsoid. Notice that in cases where $W_0 = U_0$ the second term vanishes. Thus, Eq. (5) is used to correct the local gravimetric quasigeoid, based on the values of the properties of the GGM, to the conventional reference value $W_{0,\,IHRF}$ by (cf. Eq. (14) in Sánchez et al. 2021a)

$$\zeta_{0,IHRF} = \frac{GM_{GGM} - GM_{GRS80}}{r_P\gamma_Q} - \frac{W_{0,IHRF} - U_{0,GRS80}}{\gamma_Q}. \tag{6}$$

$GM_{GRS80} = 3.986005 \cdot 10^{14}$ m^3s^{-2} is the geocentric gravitational constant of the GRS80 reference ellipsoid, r_P is the geocentric radial distance of the point P, $U_{0,\,GRS80} = 62{,}636{,}860.850$ m^2s^{-2} is the normal potential at the GRS80 reference ellipsoid. γ_Q is the normal gravity at a point P, with latitude φ, and height Q above the ellipsoid (cf. Eq. (2-215), Hofmann-Wellenhof and Moritz 2016). It can be expressed as

$$\gamma_Q = \gamma_0\left[1 - \frac{2}{a}\left(1 + f + m - 2f\sin^2\varphi(P)\right)h_Q + \frac{3}{a^2}h_Q^2\right], \tag{7}$$

where $h_Q = h(P) - \zeta(P)$, i.e. the ellipsoidal height h at point P minus the interpolated height anomaly ζ at point P, γ_0 is the normal gravity at the ellipsoid for the same latitude,

$$\gamma_0 = 9.780327 \times \left(1 + 0.0053024\sin^2\varphi - 0.0000058\sin^4\varphi\right)\left[ms^{-2}\right]. \tag{8}$$

The GRS80 parameters (cf. Moritz 2000) are *the* semi-major axis, $a = 6378137\ m$, the flattening, $f = 0.0035281068118$, and the unitless geodetic parameter $m = 0.00344978600308$.

The GRAVSOFT (Tscherning et al. 1992) program GEOCOL17, which is used in the computation of the FOGEOID10 and GGEOID16, uses $U_{0,\,GRS80}$ as reference ellipsoid and gravity potential on the geoid, i.e. $W_0 = U_0$. Thus a ζ_0 is already part of the mentioned gravimetric quasigeoids (and the same is the case for the GGMs which are used for comparison). This ζ_0 is then converted to the IHRF reference value by

$$\zeta_{0,IHRF} = -\frac{W_{0,IHRF} - U_{0,GRS80}}{\gamma_Q} \tag{9}$$

Having applied this correction, we obtain correct $\zeta(P)$ and we can compute the provisional gravity potential value at P (cf. Eqs. (8) and (9) in Sánchez et al. 2021b):

$$W_{prov}(P) = W_0 - (h(P) - \zeta(P))\,\overline{\gamma}_{QQ_0}, \tag{10}$$

Where

$$\overline{\gamma}_{QQ_0} = \gamma_0 \left(1 - \frac{1}{a}\left(1 + f + m - 2f\sin^2\varphi(P)\right) h_Q\right), \tag{11}$$

is the mean normal gravity between the point Q on the telluroid and the reference ellipsoid.

$$\overline{\gamma}_{QQ_0} = \gamma_0 \left(1 - \frac{1}{a}\left(1 + f + m - 2f\sin^2\varphi(P)\right) h_Q\right), \tag{12}$$

is the mean normal gravity between the point Q on the telluroid and the reference ellipsoid.

It is recommended to compute the geopotential values in the tide-free or zero-tide system and then afterwards transform into a mean-tide system, which is the convention of the IHRF (Sánchez et al. 2021a).

3.2 Permanent Tide Transformations

The IHRS conventions state that the all parameters in the IHRF should be in the mean-tide system. The reason is to support the geodetic monitoring of geophysical phenomena governed by fluids, e.g. oceanographic and hydrographic, which are best represented by a mean-tide system.

Both zero-tide and tide-free systems are being used for Earth's gravity field and crustal modelling, albeit that the IAG Resolution No. 16, 1983 (Tscherning 1984) states the zero-tide system should be used. There are two parts to consider in the permanent tide system transformation: The solution of the GBVP and the ITRF coordinates.

If the solution of the GBVP is in the zero-tide system a correction W_{T0} is necessary to transform $W_{prov}(P)$ to the mean-tide system. In that case we need the temporal average W_{T0} of the tide-generating potential. Written as function of geodetic latitude, and using that the geoid deviates from the GRS80 ellipsoid by a maximum of 100 m we have (Ihde et al. 2008):

$$W_{T0} = 0.9722 - 2.8841\sin^2\varphi - 0.0195\sin^4\varphi \,\left[m^2 s^{-2}\right]. \tag{13}$$

Geopotential numbers in the mean-tide system are defined as (cf. Eq. (19) Sánchez et al. 2021a)

$$C_{MT} = W_0 - (W_{ZT} + W_{T0})\,, \tag{14}$$

where $W_{ZT} = W_{prov}$. If the solution of the GBVP is in the tide-free system, we need an additional correction, to first transform from the tide-free system to the zero-tide system, and then transform to the mean-tide system. This can be done by adding the correction (cf. Eq. (38) Mäkinen 2021):

$$\Delta\overline{W}^{GGM}(\varphi, h) = k_{20}\left(1 - \tfrac{3h}{a}\right) \times \left(0.9722 - 2.8673\sin^2\varphi - 0.0690\sin^4\varphi\right)\left[m^2 s^{-2}\right], \tag{15}$$

where $k_{20} = 0.30190$ is the conventional Love number.

ITRF position coordinates are conventionally given in the tide-free system and it is necessary to restore the effects on the potential removed with the permanent tide. Another tide system related correction to W_{prov} must be added (cf. eq. (26) Mäkinen 2021):

$$\Delta W^{ITRF} \approx -0.5901 + 1.7475\sin^2\varphi + 0.0273\sin^4\varphi\,\left[m^2 s^{-2}\right]. \tag{16}$$

In the case of both the solution of the GBVP and the ITRF coordinates being in the tide-free system we get

$$W_{ZT} = W_{prov} + \Delta\overline{W}^{GGM} + \Delta W^{ITRF} \tag{17}$$

and then use Eq. (14) to compute the geopotential number $C(P)$. With the geopotential number we can determine the physical height, e.g. the normal height, H^*, of the station in the IHRF with the relation

$$H^* = \frac{C}{\overline{\gamma}_{QQ_0}} \tag{18}$$

3.3 Determining Geopotential Numbers at the IHRF Densifications in Denmark, The Faroe Islands and Greenland

The computation of the geopotential numbers at the IHRF densifications for the three countries in the Kingdom of Denmark follows a similar process, but with some small differences between the three cases. The general scheme can be seen in Fig. 4, where the different steps are listed. We recommend this workflow in determining the geopotential numbers. Some of the first steps regarding the stations and the quasigeoid can be done simultaneously, but others

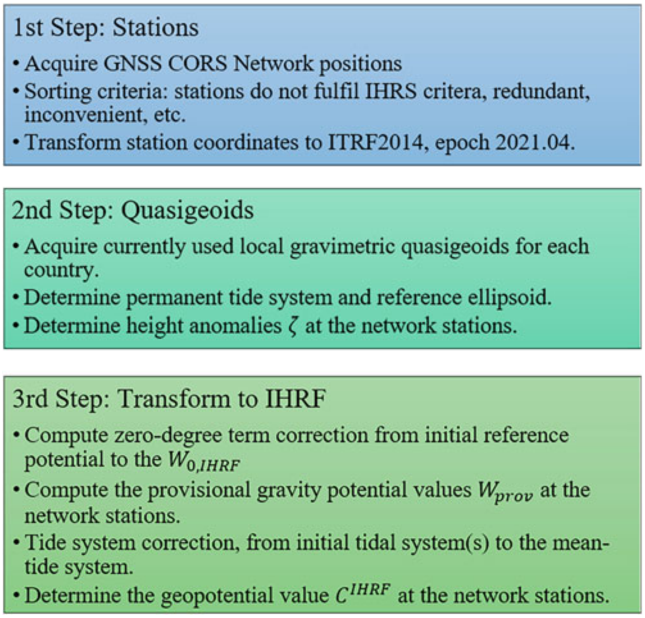

Fig. 4 General working scheme to determine geopotential numbers, going from a national height system to the International Height Reference Frame. The steps in the scheme are a recommended workflow

require previous steps to be completed. To determine the geopotential numbers the position of the densifications in the ITRF2014 epoch 2021.04 was determined. Next, the height anomaly at each densification was determined by interpolation at the location. Table 3 shows the different corrections and transformations applied in the step "Transform to IHRF" in Fig. 4, for the three countries in the Kingdom of Denmark, along with an average of the total correction and a standard deviation. The actual correction at each station depends on the height and latitude of the station.

4 Results and Analysis

The full results of the densification of the IHRF for the 14 stations in Denmark, the 4 stations in The Faroe Islands and the 59 stations in Greenland are listed in the tables in the Appendix. It includes the station name, the ITRF2014, epoch 2021.04, position in ellipsoidal coordinates, the height anomalies interpolated from the local gravimetric quasigeoid, and the IHRF normal height, geopotential number and potential, inferred from the station coordinates and the height anomalies.

4.1 Offset from Local Height Systems to IHRF

The mean offset between the national height systems and the normal heights in the IHRF, is determined as the mean of the difference between the normal heights in the two systems. The normal heights H^* in the national systems are computed as the difference between the local height anomaly ζ from the gravimetric quasigeoid models and the ellipsoidal height h.

$$H^* = h - \zeta \tag{18}$$

The normal heights in the IHRF are computed according to Eq. (18). Table 2 shows the results from computing the differences between the IHRF geopotential numbers and the local height systems in Denmark, The Faroe Islands and Greenland, respectively, in centimetres. The first row shows the difference to between the gravimetric quasigeoid used for computing the IHRF coordinates and the local height system, the second row shows the mean of the transformation correction from the local heights to IHRF heights, and the third row shows the final mean difference between the local height systems and the IHRF. These biases are the offsets, with corresponding uncertainties between the current local physical height systems in the countries and the IHRF, and produce the link, from said height systems, to all height systems connected to the IHRF. The results in the table indicate the differences in offsets between local height systems and the IHRF. This offset is very much dependent on the location and on the definition of the local height system (Table 3).

4.2 Comparison to EGM2008 and XGM2019

For the sake of comparison and validation of the results of the IHRF densification the height anomalies, ζ, at the DK-net, FO-net and GNET stations were computed with the global geopotential models of EGM2008 to d/o 2,160 and XGM2019e to d/o 719. The geopotential numbers in the IHRF were computed following the same scheme in the method section, using Eq. (9) to compute the ζ_0-correction, accounting for the different permanent tide-systems.

Finally, H^* at the densifications is computed from the geopotential number in the IHRF and subtracted from the corresponding H^* from the local gravimetric quasigeoids. Statistics of the comparison in the three countries can be seen in Table 4, which shows the mean, the standard deviation and the extrema. Overall the average differences are within ± 10 cm, but the standard deviations also show that there is a big variation in the differences, especially in Greenland. For the DK-net the GGMs agree well with the local gravimetric quasigeoid. The statistics of the differences, are quite close for the two cases. The standard deviation shows that there is a reasonably variation across the densifications. This small difference can be due to the topography in the region being quite flat, with high quality gravity data coverage.

Table 2 The biases, with corresponding uncertainties, between the local height systems in Denmark, The Faroe Islands, and Greenland, the gravimetric quasigeoids, and the IHRF conventional W_0. The mean of the full transformation to IHRF is also included. All units are in centimetres

Height system	DVR90	FVR09	GVR16
Bias to gravimetric quasigeoid	−48.00 ± 1.70 cm	−7.00 ± 5.00 cm	15.60 ± 10.00 cm
Mean of full IHRF transformation correction	−3.98 ± 0.15 cm	−66.77 ± 0.09 cm	−69.70 ± 0.71 cm
Bias from height system to IHRF	−44.20 ± 1.71 cm	−59.77 ± 5.00 cm	−54.10 ± 10.03 cm

Table 3 Correction to ζ_0, tide system transformation and average of the total correction, of the corresponding normal height, for the three different countries in the Kingdom of Denmark

	NKG2015quasigeoid	FOGEOID2010	GGEOID16
$\boldsymbol{\zeta_0}$ ***correction***	Not necessary	$-\frac{W_{0,IHRF}-U_{0,GRS80}}{\gamma_Q}$	$-\frac{W_{0,IHRF}-U_{0,GRS80}}{\gamma_Q}$
Tide system transformation	$\Delta W^{ITRF} + W_{T0}$	$\Delta\overline{W}^{GGM} + \Delta W^{ITRF} + W_{T0}$	$\Delta W^{ITRF} + W_{T0}$
Average of total correction, in $\boldsymbol{H^*}$	−3.98 ± 0.15 cm	66.80 ± 0.09 cm	69.65 ± 0.71 cm

Table 4 The statistics of the difference in normal height, inferred from C^{IHRF} from the Nordic-Baltic NKG2015 gravimetric quasigeoid, the Faroese FOGEOID2010, and the Greenlandic GGEOID16, to the corresponding normal heights from the global geopotential models EGM2008 and XGM2019, respectively

	Mean [cm]	St.dev. [cm]	Minimum [cm]	Maximum [cm]
NKG2015quasigeoid				
ΔEGM2008	−2.13	6.29	−11.80	8.90
ΔXGM2019	−1.82	6.28	−10.40	6.30
FOGEOID2010				
ΔEGM2008	−5.30	2.40	−7.60	−3.00
ΔXGM2019	−9.30	4.90	−14.20	−5.00
GGEOID16				
ΔEGM2008	7.40	28.40	−52.80	95.20
ΔXGM2019	3.10	25.10	−37.70	87.50

In the Faroe Islands the differences are ranging from −5.3 cm to −9.3 cm for FOGEOID2010, with standard deviations below 5 cm, though it should be noted that the statistical basis in the comparison is weak, with only four stations being compared. EGM2008 was used in the determination of the FOGEOID2010, and so it could be expected to give very similar results.

For the GNET stations the mean difference is 7.4 cm and 3.1 cm for the EGM2008 and XGM2019, respectively. The spread of the differences is quite a lot larger than for other countries, up to 28.4 cm. The differences are smaller for XGM2019 than for EGM2008, which could be due to the inclusion of more satellite data in the EIGEN-6C4, which was used in the determination for the gravimetric quasigeoid. The large differences could have been expected, because Greenland is the largest and most rugged country of the three.

5 Discussion

5.1 Estimated Accuracy of the IHRF Geopotential Numbers

The result of the comparison to the GGMs is an attempt at determining the accuracy of the densification networks. All mean differences are within ±10 cm. The largest difference in Denmark and Greenland is to the EGM2008, while the difference to XGM2019 is largest in the Faroe Islands. The XGM2019 includes a substantial amount of satellite gravity data that was not available for the EGM2008. The GGMs used in NKG2015 and GGEOID16 also include much of the same satellite gravity data, e.g. data from the GOCE mission. Apart from that they should be based on largely the same terrestrial data for the three countries, as XGM2019 up to d/o 719 is based on 15′ gravity anomalies from NGA, which likely also have gone into the making of the EGM2008.

The question is then how accurate the inferred IHRF geopotential numbers are. The gravimetric quasigeoids are all modelled with the goal of optimising for respective regions that they cover. All three regions are to a large extent well covered by gravity data, and the comparison of the gravimetric quasigeoids to local GNSS-levelling data is mostly good. It would thus be expected that the geopotential numbers and physical heights inferred from the local gravimetric quasigeoids would have the advantage in accuracy over the corresponding results from the EGM2008 and XGM2019, and a careful estimate would be an accuracy on the same order as the local gravimetric quasigeoids used in the inference of the geopotential numbers. And this is also supported by the results of the comparison to EGM2008 and XGM2019.

Possibly the most delicate part of determining the geopotential numbers in IHRF is to figure out the correct way of determining the ζ_0 term or the correction to apply. Already existing national/regional geoids or quasigeoids are modelled and determined by different experts. Since there is not just a single solving the GBVP these models are constructed in different systems, with different references and conventions. Thus, some of the models might have a ζ_0 referring to one reference, while another model refers to different one. If one then strictly follows the method in Sánchez et al. (2021a) or (2021b) and determines the ζ_0 by Eq. (6) the inferred geopotential numbers will refer to a wrong W_0. In the current three cases we have exactly this problem that they cannot be treated in the same way. The ζ_0 of the NKG2015 gravimetric quasigeoid is already referring to $W_{0,\,IHRF}$. On the other hand, the FOGEOID10 and the GGEOID16 have a ζ_0 referring to the $U_{0,\,GRS80}$, and thus the ζ_0 has to be determined by Eq. (9). It is therefore essential, in the inference of the geopotential numbers in the IHRF, to be well informed on local geoid/quasigeoid models which are used, as it is otherwise easy to get erroneous results.

5.2 Outlook

The first IHRF solution is established, and these densifications contribute to the local establishments in Denmark, The Faroe Islands and Greenland. The densifications do not have the overall accuracy of the stationary coordinates (±0.003 m) as targeted by the IHRS working group (Sánchez et al. 2021a), since the local gravimetric quasigeoid models don't have this accuracy. Rather the accuracy of the gravimetric quasigeoids is around an order of magnitude higher. However, this is a good target to work towards with future updates of the IHRF network and densifications.

With regards to the densifications in this study, there will certainly be possibilities of improvements in the near future. With the finalization of the FAMOS project and the release of the Baltic Sea Chart Datum 2000 (Schwabe et al. 2020, 2023; Liebsch et al. 2023), there will be a new and more accurate gravimetric quasigeoid for the Nordic-Baltic region, which in turn should result in more accurate stationary coordinates for the densifications in Denmark. Likewise, a new gravimetric quasigeoid of Greenland is under development at DTU Space. It will include new airborne gravity data, as well as updated topographic data, an updated ice thickness model, etc., which should result in an improved accuracy for the core IHRF network stations and the densifications in the region as well. As for the Faroe Islands, there is certainly possibilities of improvement. The FOGEOID2010 dates back to 2010 and more than a decade's worth of satellite gravity data is collected since then. Thus, a new and improved gravimetric quasigeoid for the Faroe Islands would likely result in an improved accuracy to the IHRF densifications in the region.

6 Conclusion

The IHRF densification is implemented for stations in Denmark, The Faroe Islands and Greenland, in accordance to the establishment of the first version of the IHRF. More specifically, IHRF coordinates are inferred for 14 CORS-GNSS stations in Denmark, 4 CORS-GNSS stations in The Faroe Islands and for 59 CORS-GNSS stations in Greenland, geographically distributed around the respective countries. Differences in the computation of the gravimetric quasigeoids, for the three countries, were regarded when inferring the geopotential numbers, specifically the correction of the ζ_0 term and the transformation of from initial tide system to the mean tide system.

Offsets from the current height systems (DVR90 in Denmark, FVR09 in The Faroe Islands, and GVR16 in Greenland) to the IHRF are on average -44.0 ± 1.9 *cm*, -59.8 ± 5.1 *cm*, and -54.1 ± 11.3 *cm* for the DVR90, FVR09 and GVR16, respectively.

Comparing to the global geopotential models EGM2008 and XGM2019 gives mean differences of up ±10 cm. This indicates that the results agree with the global models, as well as there being a gain in accuracy from using the local gravimetric quasigeoid models.

Finally, the IHRF densification in Denmark, The Faroe Islands and Greenland provide the ability to link the local height systems to a global height system and to height systems in the neighbouring countries in Europe and North America.

Appendix: IHRF Coordinates of the Densifications

Tables 5, 6 and 7

Table 5 The table shows the result of the IHRF densification in Denmark, using the gravimetric quasigeoid NKG2015quasigeoid

Station	ITRF2014, epoch 2021.04			NKG2015quasigeoid, zero-tide	IHRF		
	Latitude, φ [°]	Longitude, λ [°]	Ellipsoidal height, h [m]	Height anomaly, ζ [m]	Normal height, H^* [m]	Geopotential number, C [m^2s^{-2}]	Potential, W [m^2s^{-2}]
BUDP	55.73902	12.50003	94.430	36.531	57.939	569.258	62636284.142
ESBC	55.49357	8.45683	59.500	41.237	18.302	179.828	62636673.572
FER5	56.52302	8.11828	67.510	40.739	26.812	263.408	62636589.992
FYHA	54.99364	9.98627	46.300	40.551	5.787	56.866	62636796.534
FYNO	55.33527	10.74251	50.660	39.336	11.363	111.649	62636741.751
GESR	54.57443	11.92292	44.600	38.544	6.093	59.878	62636793.522
GREJ	55.75866	9.57119	136.220	40.373	95.887	942.103	62635911.297
HABY	55.97184	11.35531	62.180	37.623	24.597	241.664	62636611.736
HIRS	57.59110	9.96755	50.590	38.766	11.867	116.568	62636736.832
MOJN	54.94431	8.80538	56.990	40.888	16.140	158.596	62636694.804
RIKO	56.12398	8.17248	48.140	41.158	7.022	68.993	62636784.407
SKEJ	56.18759	10.17984	110.860	39.432	71.469	702.151	62636151.249
SUL5	56.84166	9.74213	120.050	39.004	81.088	796.587	62636056.813
TEJH	55.24842	14.83932	41.560	34.915	6.684	65.673	62636787.727

Table 6 The table shows the result of the IHRF densification The Faroe Islands, using the gravimetric quasigeoid FOGEOID2010

Station	ITRF2014, epoch 2021.04			FOGEOID10, tide-free	IHRF		
	Latitude, φ [°]	Longitude, λ [°]	Ellipsoidal height, h [m]	Height anomaly, ζ [m]	Normal height, H^* [m]	Geopotential number, C [m^2s^{-2}]	Potential, W [m^2s^{-2}]
ARGI	61.99737	−6.78351	110.320	56.287	53.365	523.828	62636329.572
KLAV	62.22636	−6.58678	68.350	56.289	11.394	111.836	62636741.564
TVOR	61.56392	−6.84591	90.900	56.006	34.225	335.973	62636517.427
VEST	62.15359	−7.14957	67.940	56.806	10.467	102.736	62636750.664

Table 7 The table shows the result of the IHRF densification in Greenland, using the gravimetric quasigeoid GGEOID16

Station	ITRF2014, epoch 2021.04			GGEOID16, zero-tide	IHRF		
	Latitude, φ [°]	Longitude, λ [°]	Ellipsoidal height, h [m]	Height anomaly, ζ [m]	Normal height, H^* [m]	Geopotential number, C [m^{2}s^{-2}]	Potential, W [m^{2}s^{-2}]
AASI	68.71932	−52.79336	56.740	22.985	33.056	324.086	62636529.314
ASKY	75.72613	−58.25734	687.480	21.401	665.388	6515.585	62630337.815
BLAS	79.53861	−22.97474	484.260	29.755	453.817	4441.475	62632411.925
DANE	74.31195	−20.19983	177.250	48.006	128.552	1259.082	62635594.318
DGJG	71.78654	−29.85020	1494.300	51.275	1442.330	14132.817	62622720.583
DKSG	76.35162	−61.67768	609.880	20.789	588.400	5761.160	62631092.240
DMHN	76.77108	−18.65568	55.690	34.676	20.324	198.982	62636654.418
GMMA	77.80942	−19.65213	521.530	30.908	489.932	4796.026	62632057.374
GROK	78.44270	−22.90376	1046.410	29.944	1015.777	9942.747	62626910.653
HEL2	66.40116	−38.21571	425.460	48.687	376.071	3688.588	62633164.812
HJOR	63.41821	−41.14787	762.940	45.960	716.274	7029.164	62629824.236
HMBG	73.67598	−28.12908	1322.760	46.476	1275.591	12494.939	62624358.461
HRDG	81.87984	−44.51735	718.690	24.859	693.144	6781.987	62630071.413
ILUL	69.24042	−51.06075	55.340	24.119	30.523	299.220	62636554.180
ISOR	65.54665	−38.97492	84.090	45.542	37.845	371.252	62636482.148
JGBL	82.20875	−31.00422	753.580	30.491	722.402	7068.034	62629785.366
JWLF	83.11165	−45.11983	112.950	24.527	87.737	858.352	62635995.048
KAGA	69.22230	−49.81463	149.890	27.297	121.895	1194.951	62635658.449
KAGZ	79.13196	−65.85296	86.460	13.786	71.986	704.553	62636148.847
KAPI	64.43235	−50.27121	103.820	30.351	72.765	713.947	62636139.453
KBUG	65.14369	−41.15756	290.960	45.071	245.186	2405.386	62634448.014
KELY	66.98742	−50.94484	230.280	30.803	198.776	1949.428	62634903.972
KMJP	83.64324	−33.37708	85.020	26.960	57.374	561.280	62636292.120
KMOR	81.25271	−63.52739	203.880	11.166	192.027	1878.984	62634974.416
KSNB	66.86328	−35.57633	1721.200	54.971	1665.528	16334.482	62620518.918

KUAQ	68.58700	−33.05276	865.310	53.876	810.735	7948.666	62628904.734
KULL	74.58063	−57.22707	94.060	21.424	71.944	704.614	62636148.786
KULU	65.57934	−37.14937	67.890	49.958	17.229	169.013	62636684.387
LBIB	75.89381	−23.85293	1483.610	40.537	1442.382	14123.681	62622729.719
LEFN	80.45668	−26.29344	691.870	33.033	658.149	6440.564	62630412.836
LYNS	64.43048	−40.19805	173.870	44.409	128.757	1263.325	62635590.075
MARG	77.18704	−65.69463	670.500	19.797	650.013	6363.641	62630489.759
MIK2	68.14029	−31.45182	815.980	56.390	758.890	7440.979	62629412.421
MSVG	72.24082	−23.91285	88.490	51.442	36.353	356.185	62636497.215
NNVN	61.63188	−44.90107	2134.240	45.601	2087.930	20496.289	62616357.111
NORD	81.60015	−16.65545	69.350	28.669	39.994	391.328	62636462.072
NRSK	79.15503	−17.72543	348.010	29.366	317.955	3111.959	62633741.441
NUUK	64.18355	−51.73117	109.350	27.717	80.928	794.081	62636059.319
PAMI	62.01157	−49.67096	99.370	31.093	67.569	663.251	62636190.149
PLPK	66.89773	−34.03347	122.300	55.970	65.629	643.645	62636209.755
QAAR	70.74041	−52.68837	52.760	26.852	25.212	247.086	62636606.314
QAQ1	60.71527	−46.04776	110.970	36.845	73.415	720.790	62636132.610
QEQE	69.25263	−53.52233	49.180	23.194	25.288	247.901	62636605.499
RINK	71.84850	−50.99398	1337.580	32.236	1304.649	12783.596	62624069.804
SCBY	80.26013	−59.59364	543.900	18.172	525.040	5138.091	62631715.309
SCOR	70.48534	−21.95034	128.930	56.248	71.986	705.521	62636147.879
SENU	61.06958	−47.14133	666.890	35.487	630.693	6191.814	62630661.586
SISI	66.93431	−53.67287	113.200	26.820	85.679	840.276	62636013.124
SRMP	72.91068	−54.39372	370.960	24.583	345.683	3386.546	62633466.854
STNO	81.60015	−16.65545	69.350	28.669	39.994	391.328	62636462.072
THU2	76.53705	−68.82506	35.830	15.907	19.233	188.305	62636665.095
TIMM	62.53554	−42.28616	313.800	44.803	268.290	2633.271	62634220.129
TIN1	68.80124	−50.46319	537.830	26.895	510.236	5002.297	62631851.103
TREO	64.27707	−41.37509	122.050	44.971	76.374	749.386	62636104.014
UPVK	72.78828	−56.12800	164.720	23.125	140.901	1380.391	62635473.009
UTMG	62.92721	−43.30642	1471.100	48.425	1421.968	13955.716	62622897.684
VFDG	70.29993	−29.81765	1293.670	53.997	1238.976	12143.461	62624709.939
WTHG	73.95520	−24.30893	1109.590	47.965	1060.932	10391.782	62626461.618
YMER	77.43289	−24.32631	1069.980	34.390	1034.900	10131.341	62626722.059

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